

First principle electronic state calculation

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$$H = \sum_{\mathbf{p}\sigma} \epsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^{\dagger} c_{\mathbf{p}\sigma} + \frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p}\sigma} \sum_{\mathbf{p}'\sigma'} V(\mathbf{q}) c_{\mathbf{p}+\mathbf{q}\sigma}^{\dagger} c_{\mathbf{p}'-\mathbf{q}\sigma'}^{\dagger} c_{\mathbf{p}'\sigma'} c_{\mathbf{p}\sigma}$$

↓ Field operator 形式

$$H = \int \hat{\psi}^{\dagger}(x) T(x) \hat{\psi}(x) dx + \frac{1}{2} \int \int \hat{\psi}^{\dagger}(x) \hat{\psi}^{\dagger}(x') v(x, x') \hat{\psi}(x') \hat{\psi}(x) dx dx'$$

↓ Green関数の導入

1 粒子 Green 関数を定義する。

$$iG(x, t, x', t') \equiv \langle N | \mathcal{T} [\hat{\psi}(x, t) \hat{\psi}^{\dagger}(x', t')] | N \rangle \\ \equiv \begin{cases} \langle N | \hat{\psi}(x, t) \hat{\psi}^{\dagger}(x', t') | N \rangle & \text{for } t > t' \\ -\langle N | \hat{\psi}^{\dagger}(x', t') \hat{\psi}(x, t) | N \rangle & \text{for } t < t' \end{cases}$$

ここで、 $\hat{\psi}^{\dagger}(x, t)$, $\hat{\psi}(x, t)$ は電子の生成消滅演算子。|N> は N 電子基底状態。

$$G(1, 2) = \text{---}\blacktriangleright\text{---} \quad v(1, 2) = \text{.....}$$

$$G_0(1, 2) = \text{---}\blacktriangleright\text{---} \quad W(1, 2) = \text{~~~~~}$$

$$\Sigma(1, 2) = \text{---}\blacktriangleright\text{---} \quad P(1, 2) = \text{---}\blacktriangleleft\text{---}$$

$$\Gamma(1, 2, 3) = \text{---}\blacktriangleright\text{---}$$

$$G(1, 2) = \text{---}\blacktriangleright\text{---} = \text{---}\blacktriangleright\text{---} + \text{---}\blacktriangleright\text{---} + \text{---}\blacktriangleright\text{---}$$

$$G(1, 2) = G_0(1, 2) + \int G_0(1, 3) \Sigma(3, 4) G(4, 2) d[34]$$

$$\Sigma(1, 2) = \text{---}\blacktriangleright\text{---} = \text{---}\blacktriangleright\text{---} + \text{---}\blacktriangleright\text{---}$$

$$\hbar \Sigma(1, 2) = i \int G(1, 3) \Gamma(3, 4; 2) W(4, 1) d[34]$$

$$W(1, 2) = \text{~~~~~} = \text{.....} + \text{---}\blacktriangleleft\text{---}$$

$$W(1, 2) = v(1, 2) + \int v(1, 3) P(3, 4) W(4, 2) d[34]$$

$$P(1, 2) = \text{---}\blacktriangleleft\text{---} = \text{---}\blacktriangleleft\text{---}$$

$$\hbar P(1, 2) = -i \int G(1, 3) \Gamma(3, 4; 2) G(4, 1) d[34]$$

$$\Gamma(1, 2, 3) = \text{---}\blacktriangleright\text{---} = \text{---}\blacktriangleright\text{---} + \text{---}\blacktriangleright\text{---}$$

$$\Gamma(1, 2; 3) = \delta(1, 2) \delta(1, 3) + \int \frac{\delta \Sigma(1, 2)}{\delta G(4, 5)} G(4, 6) \Gamma(6, 7; 3) G(7, 5) d[4567]$$

Hedin's equations

$$G(1, 2) = G_0(1, 2) + \int G_0(1, 3) \Sigma(3, 4) G(4, 2) d[34]$$

$$\hbar \Sigma(1, 2) = i \int G(1, 3) \Gamma(3, 4; 2) W(4, 1) d[34]$$

$$\hbar P(1, 2) = -i \int G(1, 3) \Gamma(3, 4; 2) G(4, 1) d[34]$$

$$\Gamma(1, 2; 3) = \delta(1, 2) \delta(1, 3) + \int \frac{\delta \Sigma(1, 2)}{\delta G(4, 5)} G(4, 6) \Gamma(6, 7; 3) G(7, 5) d[4567]$$

$$W(1, 2) = v(1, 2) + \int v(1, 3) P(3, 4) W(4, 2) d[34]$$